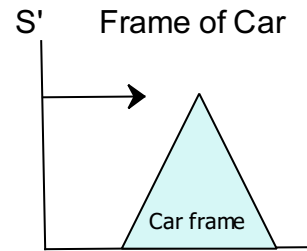
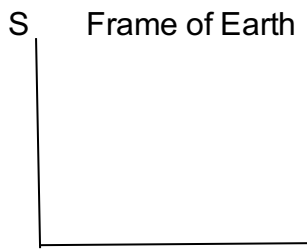


Relativistic physics Lorentz transformation

What is the Lorentz contraction of a automobile traveling at 60 mph? (60 mph is equivalent to 2682 cm/sec.)



Solution

Suppose we are given two frames of reference that are moving relative to one another with a velocity of v . If we are dealing with classical physics and want to relate the coordinates of an event occurring in the S-frame (x, y, z, t) to the coordinates of an event occurring in the S'-frame (x', y', z', t'), we use the Galilean transformation, or

$$\begin{aligned}x' &= x - vt \\y' &= y \quad (\text{if } v \text{ is in the } x\text{-direction only}) \\z' &= z \\t' &= t\end{aligned}$$

In relativistic physics, this transformation is invalid and must be replaced by the Lorentz transformation, or

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$\begin{aligned}y' &= y \\z' &= z\end{aligned}$$

$$t' = \frac{t - v \frac{x}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Now, we may relate distance measured in (S')s to the distance measured in (S)s. Let us imagine the measurement of distance which is parallel to the x'-axis in the S' frame. In order to measure the length of a rod in S, we must locate both ends of the rod (x_1, x_2) at the same time ($t_1 = t_2$) in S. The length in S' is

$$x_2' - x_1' = \frac{(x_2 - x_1) - v(t_2 - t_1)}{\sqrt{1 - v^2/c^2}}$$

$$x_2' - x_1' = \frac{(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

$$x_2 - x_1 = (x_2' - x_1') \cdot \sqrt{1 - v^2/c^2} \quad (1)$$

$$\sqrt{1 - v^2/c^2} < 1 \quad \longrightarrow \quad x_2 - x_1 < x_2' - x_1'$$

The observer in S measures a smaller rod length (which is contracted) than the observer in the rod's rest frame S'. We calculate the length of the car in S, ($x_2 - x_1$).

$$V_r := 26822.4 \text{ mm/s}$$

$$\left(\frac{V_r}{c}\right)^2 = 8.004e-9 \text{ mm}^2 \text{ s}^{-2} \text{ s}^2 \text{ m}^{-2}$$

When x is much less than 1, $\sqrt{1-x} = 1 - \frac{1}{2} \cdot x$ approximately.

$$\sqrt{1 - \left(\frac{V_r}{c}\right)^2} = 1 - (4.0 \text{ e} - 15)$$

$$x_2 - x_1 = (x_2' - x_1') \cdot (1 - 4.0 \text{ e} - 15)$$

This means that the change in length of a meter rule is only 4.0×10^{-15} meters, or 4.0×10^{-13} cm. Since

the diameter of an atom is about 10^{-8} cm, the diameter of a nucleus is about 10^{-12} cm and the size of the electron is about 10^{-13} cm, this contraction is clearly negligible. Again we see that the difference between relativistic and classical physics is not important for the velocities we are normally concerned with.