Relativistics physics Lorentz transformation

What is the Lorenz contraction of a automobile traveling at 60 mph? (60 mph is equivalent to 2682 cm/sec.)

Solution

Suppose we are given two frames of reference that are moving relative to one another with a velocity of v. If we are dealing with classical physics and want to relate the coordinates of an event occurring in the S-frame (x, y, z, t) to the coordinates of an event occurring in the S'-frame (x', y', z', t'), we use the Galilean transformation, or

$$x' = x - v\,t$$
$$y' = y \qquad \text{(If v is in the x-direction only)}$$
$$z' = z$$
$$t' = t$$

In relativistic physics, this transformation is invalid and must be replaced by the Lorentz transformation, or

$$x' = \frac{x - v\,t}{\sqrt[2]{1 - v^2 / c^2}}$$

$$y' = y$$
$$z' = z$$

$$t' = \frac{t - v\dfrac{x}{c^2}}{\sqrt[2]{1 - v^2 / c^2}}$$

Now, we may relate distance measured in (S')s to the distance measured in (S)s. Let us imagine the measurement of distance which is parallel to the x'-axis in the S' frame. In order to measure the length of a rod in S, we must locate both ends of the rod (x_1,x_2) at the same time $(t_1 = t_2)$ in S. The length in S' is

$$x_2' - x_1' = \frac{(x_2 - x_1) - v\,(t_2 - t_1)}{\sqrt[2]{1 - v^2 / c^2}}$$

$$x_2' - x_1' = \frac{(x_2 - x_1)}{\sqrt[2]{1 - v^2 / c^2}}$$

$$x_2 - x_1 = (x_2' - x_1') \cdot \sqrt[2]{1 - v^2 / c^2} \qquad (1)$$

$$\sqrt[2]{1 - v^2 / c^2} < 1 \quad \Longrightarrow \quad x_2 - x_1 < x_2' - x_1'$$

The observer in S measures a smaller rod length (which is contracted) than the observer in the rod's rest frame S'. We calculate the length of the car in S, $(x_2 - x_1)$.

$$V_r := 26822.4\ \mathrm{mm/s}$$

$$\left(\frac{V_r}{c}\right)^2 = 8.004\mathrm{e}{-9}\ \mathrm{mm}^2\ \mathrm{s}^{-2}\ \mathrm{s}^2\ \mathrm{m}^{-2}$$

When x is much less than 1, $\sqrt[2]{1 - x} = 1 - \frac{1}{2} \cdot x$ approximately.

$$\sqrt[2]{1 - \left(\frac{V_r}{c}\right)^2} = 1 - (4.0\ \mathrm{e} - 15)$$

$$x_2 - x_1 = (x_2' - x_1') \cdot (1 - 4.0\ \mathrm{e} - 15)$$

This means that the change in length of a meter rule is only 4.0×10^{-15} meters, or 4.0×10^{-13} cm. Since the diameter of an atom is about 10^{-8} cm, the diameter of a nucleus is about 10^{-12} cm and the size of the electron is about 10^{-13} cm, this contraction is clearly negligible. Again we see that the difference between relativistic and classical physics is not important for the velocities we are normally concerned with.