Particles collision

A particle with rest mass of M is moving with a speed(v_0) collides with and sticks to a stationary particle with a rest mass of m. What is the speed of the composite system after the collision?

Solution

In fact, we are asked to find the speed of the center of mass of the system, since it coincides with the composite particle after the collision. The velocity, momentum and total energy of a relativistic particle are related to each other as

$$\frac{v}{c} := \frac{P \, c}{E}$$

We can define the center of mass of a composite system as

$$\frac{v_{Cm}}{c} := \frac{\sum\limits_{i=1}^{n} (P_i \, c)}{\sum\limits_{i=1}^{n} E_i}$$

Since there are no other forces acting on the system, the velocity of the center of mass does not change during the collision. Before the collision, we have

$$P := 0 \qquad\qquad E := m \, c^2$$

$$P_M := M \, \gamma_0 \, v_0 \qquad\qquad E_M := M \, c^2 \cdot \gamma_0$$

$$\gamma_0 := \frac{1}{\sqrt[2]{1 - \dfrac{v_0^2}{c^2}}}$$

$$\frac{v_{Cm}}{c} := \frac{P_M \, c}{E + E_M}$$

$$v_{Cm} := \frac{M \, \gamma_0}{m + M \, \gamma_0} \cdot v_0$$