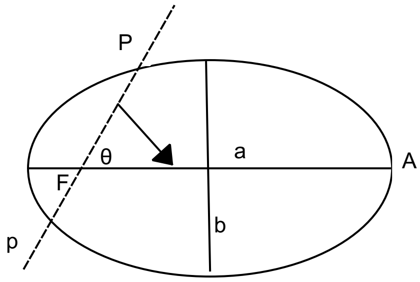
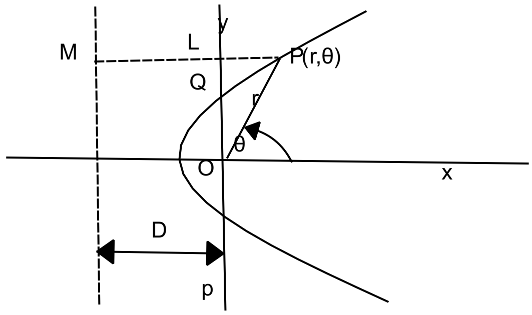
Mechanical linkage

An elliptic cam rotates about its focus, F, causing a roller at P to move up and down parallel to the y-axis. If the diameters of the cam are six and ten inches and it rotates at the rate of 300 r.p.m., how fast is the roller moving at the moment when the long axis of the cam creates an angle of 60^o with the line of motion for the roller?





Solution:

In solving this problem, we take in consideration some general ideas about conics:

If a point P(r,θ) moves so that its distance from a fixed point (called the focus), divided by its distance from a fixed line (called the directrix), is a constant E (called the eccentricity), then the curve described by P is called a conic (so-called because such curves can be obtained by cutting a cone at different angles) .

If the focus is chosen at the origin which is 0, the equation of a conic in polar coordinates (r,θ) is, for OQ = P and
LM = D,

$$F := \frac{P}{1 - E \cdot \cos(\theta)} \qquad F := \frac{e \cdot D}{1 - E \cdot \cos(\theta)}$$

Which one of the four possible conics we have depends on e ?, as follows:

1. E = 0 for a circle,
2. E < 1 an ellipse,
3. E = 1 a parabola, and
4. E > 1 a hyperbola.

For the ellipse in question, the relation between P and θ is expressed as

$$P := \frac{b^2}{a \cdot (1 - E \cdot \cos(\theta))} \quad \text{(a)}$$

In the above equation, b and e are constants. We find dp by writing the equation (a) as

$$P := \frac{b^2}{a} \cdot \left(\frac{1}{1 - E \cdot \cos(\theta)} \right)$$

$$dp := \frac{b^2}{a} \cdot \frac{d}{d\theta} \frac{1}{1 - E \cos(\theta)}$$

and by the formula for derivative,

$$dp = \frac{-E\ a^{-1}\ b^2\ \sin(\theta)}{E^2\ \cos(\theta)^2 - 2\ E\ \cos(\theta) + 1}$$

$$P_t := dp\ \theta_t \qquad \text{(b)}$$

Since half the long diameter a = 5, half the short diameter b = 3, and θ_t = 300 r.p.m. = 5 rev./sec. One full revolution sweeps 2π radians, hence θ_t = 10π rad./sec. Using these values, equation (b) for any angle θ becomes:

$$a := 5 \qquad b := 3 \qquad \theta_t := 10\ \pi$$

$$E := \frac{\sqrt[2]{a^2 - b^2}}{a} \qquad Pt = \frac{-\theta t\ E\ a^{-1}\ b^2\ \sin(\theta)}{E^2\ \cos(\theta)^2 - 2\ E\ \cos(\theta) + 1}$$

While 0 ≤ θ ≤ π, p is decreasing, and therefore, the roller is moving down. At θ = π the velocity is zero. p = 2a - FA, which is the lowest position of the roller.

Now, we wish to find the instantaneous rate of motion of the roller at θ = 60^0 = π/3 rad. At this moment,

$$P := \frac{b^2}{a} \cdot \left(\frac{1}{1 - E \cdot \cos(\theta)} \right)$$

$$dp := \frac{b^2}{a} \cdot \frac{d}{d\theta} \frac{1}{1 - E \cos(\theta)}$$

$$P_t := dp \; \theta_t$$

$$dp = \frac{-1.440 \sin(\theta)}{1 - 1.6 \cos(\theta) + 0.640 \cos(\theta)^2}$$

$$Pt = \frac{-45.239 \sin(\theta)}{1 - 1.6 \cos(\theta) + 0.640 \cos(\theta)^2}$$

Therefore, the roller is moving downward at a rate of 108.8 in/sec or 9.07 ft/sec.

$$dp := \text{replace symbols}(dp \; , \theta \; , \pi/3 \;)$$

$$P_t := dp \; \theta_t$$

$$Pt = -108.828$$