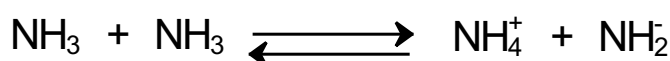


Acid base equilibria 1

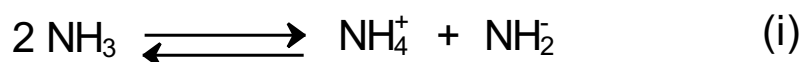
Indicate the equilibrium equation and the constant expression for the auto-protolysis of liquid ammonia. If $K_{\text{NH}_3} = 10^{-22}$, how many molecules of ammonia are ionized in a single mole of ammonia? Assume the density of ammonia is 0.771 g/ml.

Solution:

Auto-protolysis is the phenomenon whereby an ammonia molecule can donate a proton to another NH_3 molecule to form a positive and a negative charged species. The equation of the auto-protolysis can be written as



or



To find the constant expression, consider the equilibrium constant expression for the reaction:

$$K_1 = \frac{(\text{NH}_4^+) \cdot (\text{NH}_2^-)}{(\text{NH}_3)^2}$$

Note that the concentration of NH_3 in pure ammonia is always constant. By the analogy of the auto-protolysis of water (where the K_w expression is written $(\text{OH}^-)(\text{H}_3\text{O}^+)$, - without $(\text{H}_2\text{O})^2$ in the denominator), the constant expression for the auto-protolysis of NH_3 is

$$K_{\text{NH}_3} = (\text{NH}_4^+) \cdot (\text{NH}_2^-) \quad (\text{ii})$$

To find the number of molecules of ammonia ionized in a single mole of ammonia, use the equation

$$K_1 = \frac{(\text{NH}_4^+) \cdot (\text{NH}_2^-)}{(\text{NH}_3)^2}$$

Let x be the number of moles of ammonia ionized. Then the NH_3 remaining non-ionized is $1 - x$ moles. Since each 2 ammonia molecules must ionize to produce a single NH_4^+ and one NH_2^- , the number of NH_4^+ is equal to the number of $\text{NH}_2^- = x/2$. Let V be the volume of one mole of ammonia. The concentration of NH_4^+ and NH_2^- can be written as $(x/2)/V_1$, and the concentration of non-ionized NH_3

is $(1 - x) / V$. The equation for K_1 can be rewritten as

$$K_1 = \frac{\left(\frac{x}{2}\right) \cdot \left(\frac{x}{2}\right)}{\left(\frac{1-x}{V_1}\right)^2} = \frac{\frac{x^2}{4}}{(1-x)^2} \cdot \frac{\frac{1}{V_1^2}}{\frac{1}{V_1^2}} = \frac{x^2}{4(1-x)^2} \quad (\text{iii})$$

X, the number of moles of ionized ammonia, can be calculated if K_1 is known. To solve K_1 , consider a more general case of the equation

$$K_1 = \frac{(\text{NH}_4^+) \cdot (\text{NH}_2^-)}{(\text{NH}_3)^2}$$

The numerator $(\text{NH}_4^+) (\text{NH}_2^-)$ is the constant expression for the auto-protolysis of NH_3 and must always equal $K_{\text{NH}_3} \cdot K_{\text{NH}_3}$ and is given as 10^{-22} .

To find $(\text{NH}_3)^2$, use the fact that the density of ammonia is 0.771 g/ml. The mole weight of ammonia is

$$\frac{0.771 \text{ g}}{17.03 \text{ g/mol}} = 0.045 \text{ kg mol kg}^{-1}$$

and the density of ammonia is 0.0453 mol/ml = 45.3 mol / liter, so $(\text{NH}_3) = 45.3 \text{ M}$. Substitute these results in

$$K_1 = \frac{(\text{NH}_4^+) \cdot (\text{NH}_2^-)}{(\text{NH}_3)^2}$$

$$K_1 := \frac{10^{-22}}{45.3^2}$$

$$K_1 = 4.873 \times 10^{-26} \quad (\text{iv})$$

Substitute this value of K_1 into (iii)

$$4.8731 \times 10^{-26} = \frac{x^2}{4(1-x)^2} \quad (\text{v})$$

To simplify the problem, note that the dissociation of ammonia is very small and $x \ll 1$. Approximate $(1-x)^2$ as 1, then (v) becomes

$$\frac{x^2}{4} = 4.873 \times 10^{-26} \quad (\text{vi})$$

Now solve to obtain $x = 4.42 \times 10^{-13}$. This is the number of moles of NH_3 that is ionized. To find the number of molecules, remember that 1 mole = 6.02×10^{23} molecules.

$$x := 4.42 \cdot 10^{-13} \cdot \text{mol} \quad y := 6.02 \cdot 10^{23} \cdot \text{molecules/mol}$$

$$X := x y$$

$$X = 266084000000.000 \text{ molecules mol mol}^{-1}$$

So, 2.66×10^{11} molecules of ammonia are ionized.