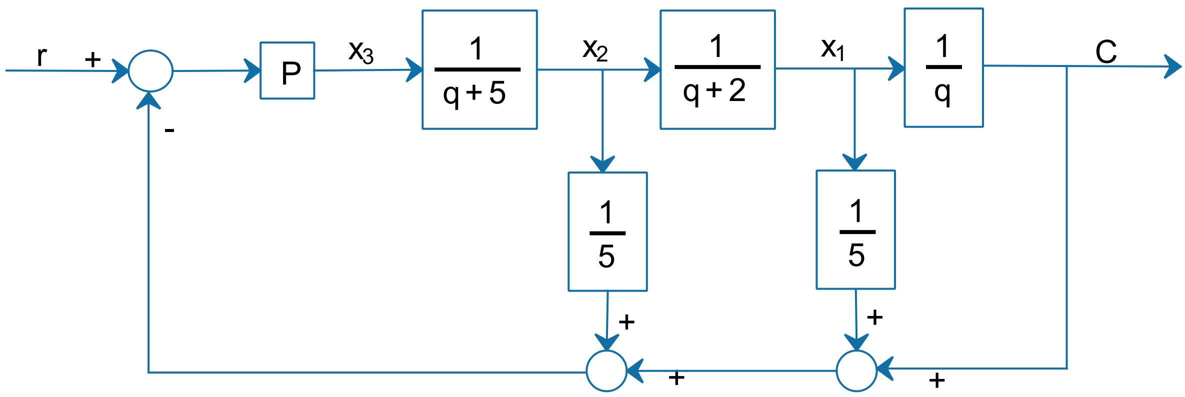
System stability

Examine the stability of the system shown in the picture below.



Solution:

We shall first find the transfer function and the characteristic equation of the system.

$$c := \frac{P}{(q+5)\cdot(q+2)\cdot q}\cdot\left(r-\left(c+\frac{1}{5}\cdot x_1+\frac{1}{5}\cdot x2\right)\right)$$

$$x_1 = cs$$

$$x_2 / (q+2) = x_1$$

$$x_2 = x_1\,(q+2) = cs\,(q+2)$$

$$M := \frac{c}{r}$$

$$M := \frac{P}{q\,(q+5)\cdot(q+2)+\frac{1}{5}\cdot P\left(q^2+3\,q+5\right)}$$

The characteristic equation of the system is

$$D_q := q\,(q+5)\cdot(q+2)+\frac{1}{5}\cdot P\left(q^2+3\,q+5\right)$$

$$D_q = q^3+7\,q^2+10\,q+0.2\,P\,q^2+0.6\,P\,q+P$$

Therefore the Routh array is:

$$a := \begin{bmatrix} q^3 & 1 & 10+0.6\,P \\ q^2 & 7+0.2\,P & P \\ q^1 & B & 0 \\ q^0 & P & 0 \end{bmatrix}$$

$$B := \frac{-1\ \text{mat determinant}(\text{subset}(a\ ,0\ ,1\ ,1\ ,2))}{\text{value at}(a\ ,1\ ,1)}$$

$$B = \frac{0.1\,P^2 + 5.2\,P + 70}{0.2\,P + 7}$$

For the system to be stable all the elements of the second column must to be positive.

$$7+0.2\,P > 0 = \begin{bmatrix} P & (-35\ ,\text{inf}\ ) \end{bmatrix}$$

$$B > 0 = \begin{bmatrix} P & (-35\ ,\text{inf}\ ) \end{bmatrix}$$

$$P > 0 = \begin{bmatrix} P & (\ 0\ ,\text{inf}\ ) \end{bmatrix}$$

When **P > 0** all the elements of the array are positive and the system is stable.