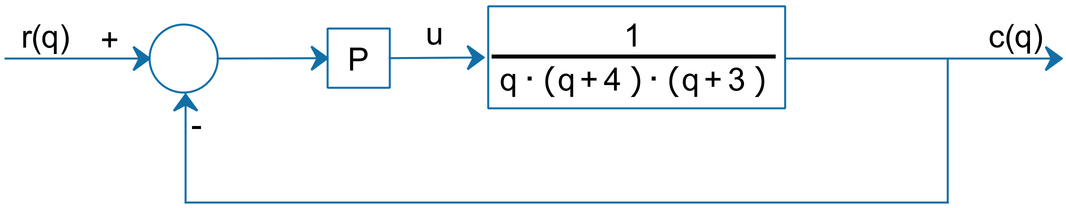
System stability

The system is shown on the figure below.
For which values of P is the system stable?



Solution:

We will find the characteristic equation of the system

$$M := \frac{c_q}{r_q}$$

$$M := \frac{P}{q\,(q+4)\cdot(q+3)+P}$$

The characteristic equation is

$$D_q := q\,(q+4)\cdot(q+3)+P$$

$$D_q = q^3 + 7\,q^2 + 12\,q + P$$

The Routh array is

$$a := \begin{bmatrix} q^3 & 1 & 12 \\ q^2 & 7 & P \\ q & \dfrac{84-P}{7} & 0 \\ q^0 & P & 0 \end{bmatrix}$$

We have a condition that the system must be stable, so all the elements of the first column must be positive.

$$\frac{84-P}{7} > 0 = \left[P \quad (-\inf, 84) \right]$$

$$P > 0 = \left[P \quad (0, \inf) \right]$$

$$P := (0, 84)$$

The system is stable if the above condition of P is satisfied.

We can define the stability margin for the parameter P.

$$\text{Stabilisy margin} := \frac{\text{maximum stable value}}{\text{actual value}}$$

Stability margin of the gain P shows us how many times it can be increased before stability occurs. In this example the value of P is 4.

$$\text{maximum stable value} := 84$$

$$\text{actual value} := 4$$

$$\text{Stabilisy margin} := \frac{\text{maximum stable value}}{\text{actual value}}$$

$$\text{Stabilisy margin} = 21$$