Root Locus 2

Sketch the root locus of

$$KGH = \frac{64\,k}{s\,(s+4)\cdot(s+16)} \qquad (1)$$

Solution:

In the equation above there are no zeros. The poles are at s = 0, s = -4, s = -16. Branches of loci start at open-loop poles and terminate at open-loop zeros at infinity.

The root locus exists on the real axis between s = 0 and s = -4, s = -16 and s = -∞. The root locus exists on the same real axis when an odd number of poles and zeros are found to the right of the point.

The asymmetrical angles are

$$\alpha = \frac{(2\,k+1)\cdot 180°}{\Sigma P\ -\ \Sigma Z}$$

$$\alpha := \frac{(2\,k+1)\cdot 180}{3}$$

$$\alpha = 120\,k+60 \qquad (2)$$

$$k := 0$$

I

$$\alpha := \frac{(2\,k+1)\cdot 180}{3}$$

$$\alpha = 60$$

$$k := 1$$

II

$$\alpha := \frac{(2\,k+1)\cdot 180}{3}$$

$$\alpha = 180$$

$$k := 2$$

III

$$\alpha := \frac{(2\,k+1)\cdot 180}{3}$$

$$\alpha = 300$$

The center of gravity is given by

$$\Sigma Z := 0 \qquad \Sigma P := 3$$

$$CG = \frac{\Sigma Pvalues - \Sigma Zvalues}{\Sigma P - \Sigma Z} \tag{3}$$

$$CG := \frac{-16 - 4 - 0 + 0}{\Sigma P - \Sigma Z}$$

$$CG = -6.667$$

The center of gravity gives the starting point for the asymptotic lines.

The breakaway point is given by

$$\frac{1}{S_b} = \frac{1}{16 - S_b} + \frac{1}{4 - S_b} \tag{4}$$

or

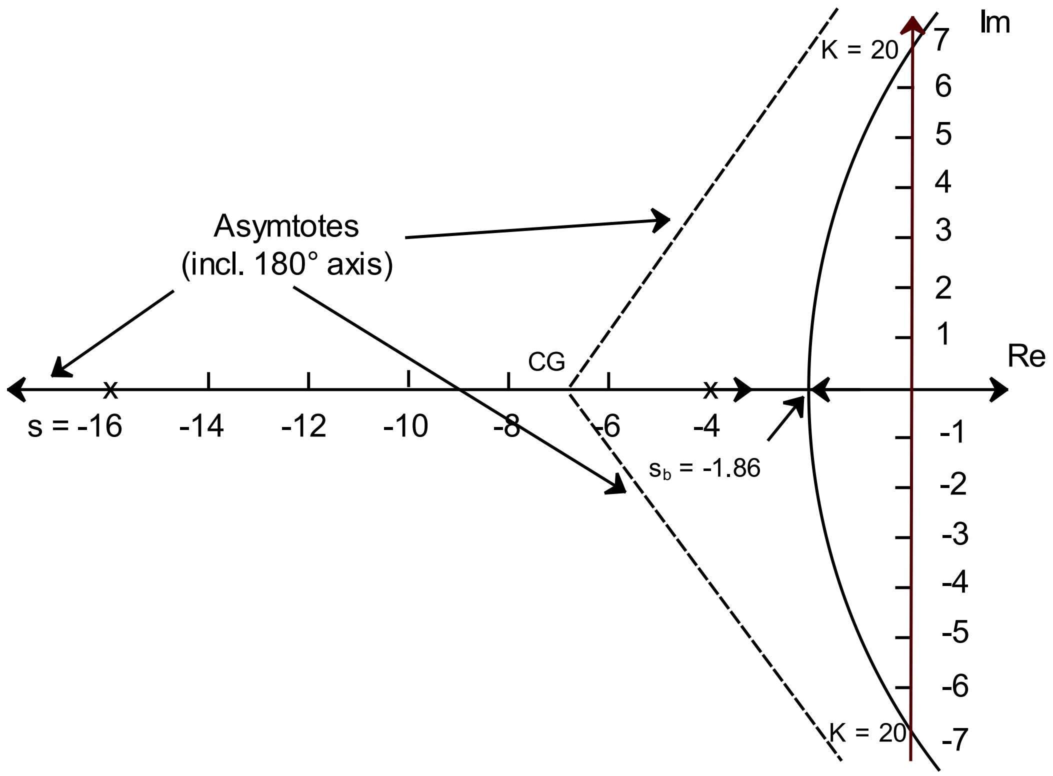
$$(16 - S_b) \cdot (4 - S_b) = S_b \cdot (4 - S_b) + S_b \cdot (16 - S_b)$$

The approximate value of S_b is

$$\text{nonlinsolve}\left((16 - S_b) \cdot (4 - S_b) == S_b \cdot (4 - S_b) + S_b \cdot (16 - S_b)\ ,\ S_b\right) = \begin{bmatrix} 1.859 & 11.474 \end{bmatrix}$$

$$S_b \approx -1.86$$

The breakaway angle from the real axis is ±90°.



The maximum value of K for which the system is stable is found by substituting s = jφ,

$$KGH(j\varphi) = \frac{64\ K}{j\varphi\,(j\varphi+4)\cdot(j\varphi+16)} \qquad (5)$$

Setting KGH (jφ) = -1 we have

$$\frac{64\ K}{j\varphi\,(j\varphi+4)\cdot(j\varphi+16)} = -1 \qquad (6)$$

Solving for K

$$K = \frac{20\varphi^2 + j\varphi\left(\varphi^2 - 64\right)}{64} \qquad (7)$$

For K to be a real number $\varphi^2 - 64$ must be equal to zero.

$$\varphi^2 == 64 = \begin{bmatrix} \varphi & \begin{bmatrix} -8 & 8 \end{bmatrix} \end{bmatrix}$$

From equation (7), for φ = ±8, we get K

$$\varphi := 8$$

$$K := \frac{20\cdot\varphi^2 + j\varphi\left(\varphi^2 - 64\right)}{64}$$

$$K = 20$$

The root locus of the system is shown in the figure below.

$$KGH = \frac{64\ K}{s\,(s+4)\cdot(s+16)}$$